Chapter 3: Forward Kinematics (I)

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Introduction and Definitions

**Robotic Manipulator:** can be modeled as a chain of rigid bodies called *links*. The links are interconnected to one another by *joints*.

One end of the chain of links is fixed to a base, while the other end is free to move (face plate with a tool or end-effector)
Introduction and Definitions

**Objective of Kinematics:** To control both the position and orientation of the tool in 3D space.

The tool can then be programmed to follow a planned trajectory so as to manipulate objects in the workspace.

In order to program the tool motion, we must first formulate the relationship between joint variables and the position and orientation of the tool !!!!

A systematic procedure for assigning coordinate frames to the links is presented (DH Algorithm) ... and this leads directly to *Arm Equation*
Introduction and Definitions

**Robot kinematics:** study of the movement of the robot referred to a reference system without considering forces and torques that cause the movement.

**Robot Kinematics** is the analytical description of spatial movement of the robot as a function of time ...

... and particularly the relationships between position and orientation of the hand, and the values of the joint variables

Robot Kinematics deals also with the relationships between speeds of joints and speeds of the end-effector
Given the vector of joint variables \((q_1, q_2, ..., q_n)^T\)

... determine the position \((x, y, z)\) and orientation (Roll, Pitch, Yaw) of the end-effector in the base coordinate system.
Introduction and Definitions

Forward Kinematics

Forward Kinematics Problem

\[(q_1, q_2, ..., q_n)^T \rightarrow (x, y, z, \theta, \phi, \alpha)^T\]

Values of Joint variables \rightarrow Position and Orientation

Is that really interesting ?????

Yes, ... but I think not at all !!

But it will be necessary for further analysis
**Introduction and Definitions**

**Inverse Kinematics**: determining the necessary joint variables given a desired position and orientation of the tool.

Manipulation tasks are naturally formulated in terms of the “desired” tool position and orientation.

**Example**: an external overhead camera provides positions and orientations of the objects (not joint variables !!!!!!!!).

That is really interesting !!!!!!!!!!

But ... we need to know Forward Kinematics
Introduction and Definitions

Forward Kinematics

1-. We will allocate a coordinate system \{Li\} for each link (*)
2-. We will compute the matrices \(i^{-1}A_i\) which transform \(L_{i-1}\) to \(L_i\)
\(i^{-1}A_i\) is a homogeneous matrix that depends of \(q_i\) (joint variable)
3-. We will compute the transformation that relates the hand with the base

\[
R_T^H = \prod_{i=1}^{n} i^{-1} A_i = 
\begin{pmatrix}
 n_x & s_x & a_x & p_x \\
 n_y & s_y & a_y & p_y \\
 n_z & s_z & a_z & p_z \\
 0 & 0 & 0 & 1
\end{pmatrix}_H
\]

The different components of this matrix are equations related to the joint variables \(q_i\).

(*) How to assign the coordinate systems to links?

**Denavit-Hartemberg Algorithm !!!**
Introduction and Definitions

$\theta$-r Manipulator RP (1st Example)

This is a robot with two joints (rotation movement $\theta_1$ about an axis, and displacement movement $d_2$ along another axis)

We place 3 coordinate systems: 1 coordinate system at the base of the robot $\{L_0\}$ and 2 coordinate systems at the ends of the links $\{L_1, L_2\}$

First link is described by the relationship between $\{L_0, L_1\}$ and the second link by the relationship between $\{L_1, L_2\}$
Introduction and Definitions

θ-r Manipulator RP (1st Example)

1-. We allocate a coordinate system \{Li\} for each link *(please, believe me !!!!)*
2-. We compute the matrices \(i^{-1}A_i\) which transform \(L_{i-1}\) to \(L_i\)
\(i^{-1}A_i\) is a homogeneous matrix that depends of \(q_i\) (joint variable)
3-. We compute \(^R_{TH} (^0T_2, \text{in this case})\)
The transformation that relates the hand with the base of the robot

Let:
\[
\begin{align*}
0 & \quad A_i \quad \text{homogeneous matrix that relates } L_1 \text{ with } L_0 \\
1 & \quad A_2 \quad \text{homogeneous matrix that relates } L_2 \text{ with } L_1
\end{align*}
\]

then

\[
^RT_H = ^0A_1 \cdot ^1A_2 = ^RT_H = \begin{bmatrix}
R & \begin{bmatrix}
  n_x & s_x & a_x & p_x \\
  n_y & s_y & a_y & p_y \\
  n_z & s_z & a_z & p_z \\
  0 & 0 & 0 & 1
\end{bmatrix}_H
\end{bmatrix}
\]

\(^RT_H\) is a matrix of equations with 2 variables: \(\theta_1\) and \(d_2\)
*(please, believe me !!!!)*
Introduction and Definitions

Forward Kinematics: Characterization of Robotic Arms

There exist a lot of possibilities to connect links and joints for obtaining poliarticulated structures:

**Angular Joints**

![Angular Joints Diagram]

**Prismatic Joints**

![Prismatic Joints Diagram]

Is it possible to obtain a general matrix $i^{-1}A_i$ that, depending of a set of parameters makes the transformation $L_{i-1}$ to $L_i$?

Yes .... we are going to see immediately !!!!
Introduction and Definitions

Forward Kinematics: Angular Joint Characterization

Imagine the general case of an Angular Joint ...

There is a well-defined methodology for doing that !!!!

Denavit-Hartememberg Algorithm

<table>
<thead>
<tr>
<th>DoF</th>
<th>θ_i</th>
<th>d_i</th>
<th>a_i</th>
<th>α_i</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

u_i is the axis of the angular joint under consideration

u_{i+1} is the axis of the next joint (angular in this case)

A coordinate system is PROPERLY attached to each link: \{L_{i-1}\} and \{L_i\}

Please ... believe me !!!!
We want to obtain $i^{-1}A_i$
Different movements we have to do from $\{L_{i-1}\}$ to $\{L_i\}$
Introduction and Definitions

Forward Kinematics: Angular Joint Characterization

<table>
<thead>
<tr>
<th>DoF</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
<th>$a_i$</th>
<th>$\alpha_i$</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>-90°</td>
<td></td>
<td></td>
<td></td>
<td>-90°</td>
</tr>
</tbody>
</table>

$A_{i-1}^{-1} = I$

$A_i^{-1} = I \cdot Rot(\theta_i, z)$

Pre o Post Multiplication? Doesn’t matter in this case!!
Introduction and Definitions

Forward Kinematics: Angular Joint Characterization

\[ i^{-1}A_i = I \]
\[ i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \]
\[ i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \]

<table>
<thead>
<tr>
<th>DoF</th>
<th>( \theta_i )</th>
<th>( d_i )</th>
<th>( a_i )</th>
<th>( \alpha_i )</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>-90(^\circ)</td>
<td>( d_i )</td>
<td></td>
<td></td>
<td>-90(^\circ)</td>
</tr>
</tbody>
</table>
Introduction and Definitions

Forward Kinematics: Angular Joint Characterization

\[
\begin{align*}
&i^{-1}A_i = I \\
&i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \\
&i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \\
&i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \cdot \text{Trans}(x, a_i)
\end{align*}
\]

<table>
<thead>
<tr>
<th>DoF</th>
<th>(\theta_i)</th>
<th>(d_i)</th>
<th>(a_i)</th>
<th>(\alpha_i)</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>-90º</td>
<td>(d_i)</td>
<td>(a_i)</td>
<td></td>
<td>-90º</td>
</tr>
</tbody>
</table>

Postmultiplication !!!
Introduction and Definitions

Forward Kinematics: Angular Joint Characterization

\[
\begin{align*}
\theta &= -90^\circ \\
da &= d_i \\
a &= a_i \\
\alpha &= \alpha_i \\
\text{Home} &= -90^\circ \\
\end{align*}
\]

\[
\begin{array}{c|c|c|c|c|c}
\text{DoF} & \theta_i & d_i & a_i & \alpha_i & \text{Home} \\
\hline
i & -90^\circ & d_i & a_i & 90^\circ & -90^\circ \\
\end{array}
\]

\[
i^{-1}A_i = I \\
i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \\
i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \\
i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \cdot \text{Trans}(x, a_i) \\
i^{-1}A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \cdot \text{Trans}(x, a_i) \cdot \text{Rot}(\alpha_i, x)
\]

Postmultiplication !!!
Introduction and Definitions

Forward Kinematics: Angular Joint Characterization

\[ i^{-1} A_i = I \]
\[ i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z) \]
\[ i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \]
\[ i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \cdot \text{Trans}(x, a_i) \]
\[ i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \cdot \text{Trans}(x, a_i) \cdot \text{Rot}(\alpha_i, x) \]

Angular Joint Characterization

\[ i^{-1} A_i = \begin{bmatrix}
  c\theta_i & -c\alpha_i \cdot s\theta_i & s\alpha_i \cdot s\theta_i & a_i \cdot c\theta_i \\
  s\theta_i & c\alpha_i \cdot c\theta_i & -s\alpha_i \cdot c\theta_i & a_i \cdot s\theta_i \\
  0 & s\alpha_i & c\alpha_i & d_i \\
  0 & 0 & 0 & 1
\end{bmatrix} \]
Introduction and Definitions

Forward Kinematics: Angular Joint Characterization

\[ i^{-1}A_i = \begin{pmatrix}
    c\theta_i & -c\alpha_i\cdot s\theta_i & s\alpha_i\cdot s\theta_i & a_i\cdot c\theta_i \\
    s\theta_i & c\alpha_i\cdot c\theta_i & -s\alpha_i\cdot c\theta_i & a_i\cdot s\theta_i \\
    0 & s\alpha_i & c\alpha_i & d_i \\
    0 & 0 & 0 & 1
\end{pmatrix}_i \]

\( \theta_i \) (is the joint angular variable): rotation about \( z_{i-1} \) needed to make axis \( x_{i-1} \) parallel with axis \( x_i \) (DEFINITION !!)

HOME POSITION: the value of the joint variable (\( \theta_i \) in this case) considering the figure, ... \( \theta_i = -\pi/2 \)
Imagine now the general case of a Prismatic Joint...

<table>
<thead>
<tr>
<th>DoF</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
<th>$a_i$</th>
<th>$\alpha_i$</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

$u_i$ is the axis of the prismatic joint under consideration

$u_{i+1}$ is the axis of the next joint (prismatic in this case)

A coordinate system is **PROPERLY** attached to each link: \( \{L_{i-1}\} \) and \( \{L_i\} \)

*Believe me again, please*

There is a well-defined methodology for doing that !!!!

**Denavit-Hartemberg Algorithm**
Introduction and Definitions

Forward Kinematics: Prismatic Joint Characterization

Prismatic Joint Characterization

\[
i^{-1} A_i = I
\]
\[
i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z)
\]
\[
i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i)
\]
\[
i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \cdot \text{Trans}(x, a_i)
\]
\[
i^{-1} A_i = I \cdot \text{Rot}(\theta_i, z) \cdot \text{Trans}(z, d_i) \cdot \text{Trans}(x, a_i) \cdot \text{Rot}(\alpha_i, x)
\]

\[
i^{-1} A_i = \\
\begin{pmatrix}
c \theta_i & -c \alpha_i \cdot s \theta_i & s \alpha_i \cdot s \theta_i & a_i \cdot c \theta_i \\
c \alpha_i \cdot c \theta_i & c \alpha_i \cdot c \theta_i & -s \alpha_i \cdot c \theta_i & a_i \cdot s \theta_i \\
0 & s \alpha_i & c \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>DoF</th>
<th>(\theta_i)</th>
<th>(d_i)</th>
<th>(a_i)</th>
<th>(\alpha_i)</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>90°</td>
<td>(d_i)</td>
<td>(a_i)</td>
<td>90°</td>
<td>(d_i)</td>
</tr>
</tbody>
</table>

Leaving out the different steps ...
Introduction and Definitions

Forward Kinematics: Prismatic Joint Characterization

<table>
<thead>
<tr>
<th>DoF</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
<th>$a_i$</th>
<th>$\alpha_i$</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$90^\circ$</td>
<td>$d_i$</td>
<td>$a_i$</td>
<td>$90^\circ$</td>
<td>$d_i$</td>
</tr>
</tbody>
</table>

It is the same matrix that the angular joint!!!

It is a General Matrix !!!!
Introduction and Definitions

Forward Kinematics: Prismatic Joint Characterization

<table>
<thead>
<tr>
<th>DoF</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
<th>$a_i$</th>
<th>$\alpha_i$</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$90^\circ$</td>
<td>$q_i$</td>
<td>$a_i$</td>
<td>$90^\circ$</td>
<td>$d_i$</td>
</tr>
</tbody>
</table>

$d_i$ (is the joint variable): translation along $z_{i-1}$ needed to make axis $x_{i-1}$ intersect with axis $x_i$ (DEFINITION !!)

HOME POSITION: the value of the joint variable ($d_i$ in this case) considering the figure, $d_i$
Introduction and Definitions

Forward Kinematics: Characterization of Robotic Arms

**Link and Joint Parameters:**

Joint angle: $\theta_k$ (joint variable in an angular joint)
Joint distance: $d_k$ (joint variable in a prismatic joint)

Link length: $a_k$
Link twist angle: $\alpha_k$
Forward Kinematics: Algorithm of Denavit-Hartenberg

Definition of a procedure to establish a relationship between the different consecutive links that compose a robotic arm

... using a set of well-defined rules to assign a reference coordinate system to each one of the links.
Forward Kinematics: Algorithm of Denavit-Hartenberg

Choosing adequately the coordinate systems attached to each link, it's possible to relate two consecutive coordinate systems using 4 basic geometrical transformations (translations and rotations) that depends exclusively of the link geometry:

1. Rotation of $\theta_i$ about the $Z_{i-1}$ axis (Joint angle: $\theta_i$)
2. Translation of $d_i$ along the $Z_{i-1}$ axis (Joint distance: $d_i$)
3. Translation of $a_i$ along the $X_i$ axis (Link length: $a_i$)
4. Rotation of $\alpha_i$ about the $X_i$ axis (Link twist angle: $\alpha_i$)

REMEMBER: angular and prismatic joint characterization !!!
Forward Kinematics: Algorithm of Denavit-Hartenberg

It’s an algorithm with two loops !!

0. Number the **joints** from 1 to \( n \) beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system \( L_0 \) to base. Axis \( Z_0 \) coincides with the axis of joint number 1. Then initialize \( k=1 \).

2. Make coinciding \( Z_k \) with axis of joint \( k+1 \).

3. Put the origin of \( L_k \) in the intersection between axis \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) don’t have intersection, use intersection of \( Z_k \) with a common normal of \( Z_k \) and \( Z_{k-1} \).

4. Put \( X_k \) orthogonal to \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) are parallels, then \( X_k \) must be perpendicular to \( Z_{k-1} \), place it along the link and pointing out.

5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1 \). If \( k<n \), goto 2.

7. Place origin of \( L_n \) at top of end-effector. Make coinciding \( Z_n \) with vector \( a \) (approach), \( Y_n \) with vector \( o \) (orientation) and \( X_n \) with vector normal \( n \) of end-effector. \( k=1 \).

8. Place the point \( b_k \) at the intersection of axis \( X_k \) and \( Z_{k-1} \). If they don’t have intersection, place this point at the intersection of \( X_k \) and a common normal to \( X_k \) and \( Z_{k-1} \).

9. \( \theta_k \) is the rotation angle from \( X_{k-1} \) to \( X_k \) measured on \( Z_{k-1} \).

10. \( d_k \) is the distance from the origin of \( L_{k-1} \) to point \( b_k \) measured along the \( Z_{k-1} \) axis.

11. \( a_k \) is the distance from point \( b_k \) to origin of \( L_k \) measured along \( X_k \) axis.

12. \( \alpha_k \) is the rotation angle from \( Z_{k-1} \) to \( Z_k \) measured on \( X_k \).

13. \( k=k+1 \). If \( k\leq n \) goto 8.

**First Loop:** locating coordinate systems  **Second Loop:** obtaining parameters
0. Number the joints from 1 to \( n \) beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system \( L_0 \) to base. Axis \( Z_0 \) coincides with the axis of joint number 1. Then initialize \( k=1 \).

2. Make coinciding \( Z_k \) with axis of joint \( k+1 \).

3. Put the origin of \( L_k \) in the intersection between axis \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) don’t have intersection, use intersection of \( Z_k \) with a common normal of \( Z_k \) and \( Z_{k-1} \).

4. Put \( X_k \) orthogonal to \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) are parallels, then \( X_k \) must be perpendicular to \( Z_{k-1} \), place it along the link and pointing out.

5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1 \). If \( k<n \), goto 2.

7. Place origin of \( L_n \) at top of end-effector. Make coinciding \( Z_n \) with vector \( a \) (approach), \( Y_n \) with vector \( o \) (orientation) and \( X_n \) with vector normal \( n \) of end-effector. \( k=1 \).

8. Place the point \( b_k \) at the intersection of axis \( X_k \) and \( Z_{k-1} \). If they don’t have intersection, place this point at the intersection of \( X_k \) and a common normal to \( X_k \) and \( Z_k \).

9. \( \theta_k \) is the rotation angle from \( X_{k-1} \) to \( X_k \) measured on \( Z_{k-1} \).

10. \( d_k \) is the distance from the origin of \( L_{k-1} \) to point \( b_k \) measured along the \( Z_{k-1} \) axis.

11. \( a_k \) is the distance from point \( b_k \) to origin of \( L_k \) measured along \( X_k \) axis.

12. \( \alpha_k \) is the rotation angle from \( Z_{k-1} \) to \( Z_k \) measured on \( X_k \).

13. \( k=k+1 \). If \( k<n \) goto 8.
DH Algorithm: Example $\theta - r$ Robot

0. Number the joints from 1 to $n$ beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system $L_0$ to base. Axis $Z_0$ coincides with the axis of joint number 1. Then initialize $k=1$.

2. Make coinciding $Z_k$ with axis of joint $k+1$.

3. Put the origin of $L_k$ in the intersection between axis $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ don’t have intersection, use intersection of $Z_k$ with a common normal of $Z_k$ and $Z_{k-1}$.

4. Put $X_k$ orthogonal to $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ are parallels, then $X_k$ must be perpendicular to $Z_{k-1}$, place it along the link and pointing out.

5. Select $Y_k$ to complete the coordinate system $L_k$.

6. $k=k+1$. If $k<n$, goto 2.

7. Place origin of $L_n$ at top of end-effector. Make coinciding $Z_n$ with vector $a$ (approach), $Y_n$ with vector $o$ (orientation) and $X_n$ with vector normal $n$ of end-effector. $k=1$.

8. Place the point $b_k$ at the intersection of axis $X_k$ and $Z_{k-1}$. If they don’t have intersection, place this point at the intersection of $X_k$ and a common normal to $X_k$ and $Z_k$.

9. $\theta_k$ is the rotation angle from $X_{k-1}$ to $X_k$ measured on $Z_{k-1}$.

10. $d_k$ is the distance from the origin of $L_{k-1}$ to point $b_k$ measured along the $Z_{k-1}$ axis.

11. $a_k$ is the distance from point $b_k$ to origin of $L_k$ measured along $X_k$ axis.

12. $\alpha_k$ is the rotation angle from $Z_{k-1}$ to $Z_k$ measured on $X_k$.

13. $k=k+1$. If $k<n$ goto 8.
DH Algorithm: Example $\theta - r$ Robot

0. Number the **joints** from 1 to $n$ beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. **Assign the coordinate system $L_0$ to base.** Axis $Z_0$ coincides with the axis of joint number 1. Then initialize $k=1$.

2. Make coinciding $Z_k$ with axis of joint $k+1$.

3. Put the origin of $L_k$ in the intersection between axis $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ don’t have intersection, use intersection of $Z_k$ with a common normal of $Z_k$ and $Z_{k-1}$.

4. Put $X_k$ orthogonal to $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ are parallels, then $X_k$ must be perpendicular to $Z_{k-1}$, place it along the link and pointing out.

5. Select $Y_k$ to complete the coordinate system $L_k$.

6. $k=k+1$. If $k<n$, goto 2.

7. Place origin of $L_n$ at top of end-effector. Make coinciding $Z_n$ with vector $a$ (approach), $Y_n$ with vector $o$ (orientation) and $X_n$ with vector normal $n$ of end-effector. $k=1$.

8. Place the point $b_k$ at the intersection of axis $X_k$ and $Z_{k-1}$. If they don’t have intersection, place this point at the intersection of $X_k$ and a common normal to $X_k$ and $Z_k$.

9. $\theta_k$ is the rotation angle from $X_{k-1}$ to $X_k$ measured on $Z_{k-1}$.

10. $d_k$ is the distance from the origin of $L_{k-1}$ to point $b_k$ measured along the $Z_{k-1}$ axis.

11. $a_k$ is the distance from point $b_k$ to origin of $L_k$ measured along $X_k$ axis.

12. $\alpha_k$ is the rotation angle from $Z_{k-1}$ to $Z_k$ measured on $X_k$.

13. $k=k+1$. If $k=n$ goto 8.
DH Algorithm: Example $\theta - r$ Robot

0. Number the **joints** from 1 to $n$ beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system $L_0$ to base. Axis $Z_0$ coincides with the axis of joint number 1. Then initialize $k=1$.

2. **Make coinciding $Z_k$ with axis of joint $k+1$.**

3. Put the origin of $L_k$ in the intersection between axis $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ don’t have intersection, use intersection of $Z_k$ with a common normal of $Z_k$ and $Z_{k-1}$.

4. Put $X_k$ orthogonal to $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ are parallels, then $X_k$ must be perpendicular to $Z_{k-1}$, place it along the link and pointing out.

5. Select $Y_k$ to complete the coordinate system $L_k$.

6. $k=k+1$. If $k<n$, goto 2.

7. Place origin of $L_n$ at top of end-effector. Make coinciding $Z_n$ with vector $a$ (approach), $Y_n$ with vector $o$ (orientation) and $X_n$ with vector normal $n$ of end-effector. $k=1$.

8. Place the point $b_k$ at the intersection of axis $X_k$ and $Z_{k-1}$. If they don’t have intersection, place this point at the intersection of $X_k$ and a common normal to $X_k$ and $Z_k$.

9. $\theta_k$ is the rotation angle from $X_{k-1}$ to $X_k$ measured on $Z_{k-1}$.

10. $d_k$ is the distance from the origin of $L_{k-1}$ to point $b_k$ measured along the $Z_{k-1}$ axis.

11. $a_k$ is the distance from point $b_k$ to origin of $L_k$ measured along $X_k$ axis.

12. $\alpha_k$ is the rotation angle from $Z_{k-1}$ to $Z_k$ measured on $X_k$.

13. $k=k+1$. If $k=n$ goto 8.
DH Algorithm: Example $\theta - r$ Robot

K=1, n=2

0. Number the **joints** from 1 to $n$ beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system $L_0$ to base. Axis $Z_0$ coincides with the axis of joint number 1. Then initialize $k=1$.

2. Make coinciding $Z_k$ with axis of joint $k+1$.

3. **Put the origin of $L_k$ in the intersection between axis $Z_k$ and $Z_{k-1}$**. If $Z_k$ and $Z_{k-1}$ don’t have intersection, use intersection of $Z_k$ with a common normal of $Z_k$ and $Z_{k-1}$.

4. Put $X_k$ orthogonal to $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ are parallels, then $X_k$ must be perpendicular to $Z_{k-1}$, place it along the link and pointing out.

5. Select $Y_k$ to complete the coordinate system $L_k$.

6. $k=k+1$. If $k<n$, goto 2.

7. Place origin of $L_n$ at top of end-effector. Make coinciding $Z_n$ with vector $a$ (approach), $Y_n$ with vector $o$ (orientation) and $X_n$ with vector normal $n$ of end-effector. $k=1$.

8. Place the point $b_k$ at the intersection of axis $X_k$ and $Z_{k-1}$. If they don’t have intersection, place this point at the intersection of $X_k$ and a common normal to $X_k$ and $Z_k$.

9. $\theta_k$ is the rotation angle from $X_{k-1}$ to $X_k$ measured on $Z_{k-1}$. 

10. $d_k$ is the distance from the origin of $L_{k-1}$ to point $b_k$ measured along the $Z_{k-1}$ axis.

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12. $\alpha_k$ is the rotation angle from $Z_{k-1}$ to $Z_k$ measured on $X_k$.

13. $k=k+1$. If $k\leq n$ goto 8.
DH Algorithm: Example $\theta - r$ Robot

0. Number the **joints** from 1 to $n$ beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system $L_0$ to base. Axis $Z_0$ coincides with the axis of joint number 1. Then initialize $k=1$.

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4. **Put $X_k$ orthogonal to $Z_k$ and $Z_{k-1}$**. If $Z_k$ and $Z_{k-1}$ are parallels, then $X_k$ must be perpendicular to $Z_{k-1}$, place it along the link and pointing out.

5. Select $Y_k$ to complete the coordinate system $L_k$.

6. $k=k+1$. If $k<n$, goto 2.

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12. $\alpha_k$ is the rotation angle from $Z_{k-1}$ to $Z_k$ measured on $X_k$.

13. $k=k+1$. If $k\leq n$ goto 8.

### DH Algorithm: Example $\theta - r$ Robot

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<th>$a_i$</th>
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$K=1, n=2$
0. Number the joints from 1 to \( n \) beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system \( L_0 \) to base. Axis \( Z_0 \) coincides with the axis of joint number 1. Then initialize \( k=1 \).

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3. Put the origin of \( L_k \) in the intersection between axis \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) don’t have intersection, use intersection of \( Z_k \) with a common normal of \( Z_k \) and \( Z_{k-1} \).

4. Put \( X_k \) orthogonal to \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) are parallels, then \( X_k \) must be perpendicular to \( Z_{k-1} \), place it along the link and pointing out.

5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1. \) If \( k<n \), goto 2.

7. Place origin of \( L_n \) at top of end-effector. Make coinciding \( Z_n \) with vector \( a \) (approach), \( Y_n \) with vector \( o \) (orientation) and \( X_n \) with vector normal \( n \) of end-effector. \( k=1 \).

8. Place the point \( b_k \) at the intersection of axis \( X_k \) and \( Z_{k-1} \). If they don’t have intersection, place this point at the intersection of \( X_k \) and a common normal to \( X_k \) and \( Z_k \).

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13. \( k=k+1 \). If \( k<n \) goto 8.
DH Algorithm: Example θ – r Robot

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4. Put \( X_k \) orthogonal to \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) are parallels, then \( X_k \) must be perpendicular to \( Z_{k-1} \), place it along the link and pointing out.

5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1 \). If \( k<n \), goto 2.

7. Place origin of \( L_n \) at top of end-effector. Make coinciding \( Z_n \) with vector \( a \) (approach), \( Y_n \) with vector \( o \) (orientation) and \( X_n \) with vector normal \( n \) of end-effector. \( k=1 \).

8. Place the point \( b_k \) at the intersection of axis \( X_k \) and \( Z_{k-1} \). If they don’t have intersection, place this point at the intersection of \( X_k \) and a common normal to \( X_k \) and \( Z_k \).

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12. \( \alpha_k \) is the rotation angle from \( Z_{k-1} \) to \( Z_k \) measured on \( X_k \).

13. \( k=k+1 \). If \( k=n \) goto 8.

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DH Algorithm: Example \( \theta - r \) Robot

0. Number the joints from 1 to \( n \) beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system \( L_0 \) to base. Axis \( Z_0 \) coincides with the axis of joint number 1. Then initialize \( k=1 \).

2. Make coinciding \( Z_k \) with axis of joint \( k+1 \).

3. Put the origin of \( L_k \) in the intersection between axis \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) don't have intersection, use intersection of \( Z_k \) with a common normal of \( Z_k \) and \( Z_{k-1} \).

4. Put \( X_k \) orthogonal to \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) are parallels, then \( X_k \) must be perpendicular to \( Z_{k-1} \), place it along the link and pointing out.

5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1 \). If \( k < n \), goto 2.

7. Place origin of \( L_n \) at top of end-effector. Make coinciding \( Z_n \) with vector \( a \) (approach), \( Y_n \) with vector \( o \) (orientation) and \( X_n \) with vector normal \( n \) of end-effector. \( k=1 \).

8. Place the point \( b_k \) at the intersection of axis \( X_k \) and \( Z_{k-1} \). If they don’t have intersection, place this point at the intersection of \( X_k \) and a common normal to \( X_k \) and \( Z_k \).

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13. \( k=k+1 \). If \( k \leq n \) goto 8.
DH Algorithm: Example θ – r Robot

0. Number the **joints** from 1 to \( n \) beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

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2. Make coinciding \( Z_k \) with axis of joint \( k+1 \).

3. Put the origin of \( L_k \) in the intersection between axis \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) don’t have intersection, use intersection of \( Z_k \) with a common normal of \( Z_k \) and \( Z_{k-1} \).

4. Put \( X_k \) orthogonal to \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) are parallels, then \( X_k \) must be perpendicular to \( Z_{k-1} \), place it along the link and pointing out.

5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1 \). If \( k<n \), goto 2.

7. Place origin of \( L_n \) at top of end-effector. Make coinciding \( Z_n \) with vector a (approach), \( Y_n \) with vector o (orientation) and \( X_n \) with vector normal n of end-effector. \( k=1 \).

8. **Place the point \( b_k \) at the intersection of axis \( X_k \) and \( Z_{k-1} \).** If they don’t have intersection, place this point at the intersection of \( X_k \) and \( Z_k \) and a common normal to \( X_k \) and \( Z_k \).

9. \( \theta_k \) is the rotation angle from \( X_{k-1} \) to \( X_k \) measured on \( Z_{k-1} \).

10. \( d_k \) is the distance from the origin of \( L_{k-1} \) to point \( b_k \) measured along the \( Z_{k-1} \) axis.

11. \( a_k \) is the distance from point \( b_k \) to origin of \( L_k \) measured along \( X_k \) axis.

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0. Number the **joints** from 1 to \( n \) beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

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3. Put the origin of \( L_k \) in the intersection between axis \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) don’t have intersection, use intersection of \( Z_k \) with a common normal of \( Z_k \) and \( Z_{k-1} \).

4. Put \( X_k \) orthogonal to \( Z_k \) and \( Z_{k-1} \). If \( Z_k \) and \( Z_{k-1} \) are parallels, then \( X_k \) must be perpendicular to \( Z_{k-1} \), place it along the link and pointing out.

5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1 \). If \( k<\n \) goto 2.

7. Place origin of \( L_n \) at top of end-effector. Make coinciding \( Z_n \) with vector \( a \) (approach), \( Y_n \) with vector \( o \) (orientation) and \( X_n \) with vector normal \( n \) of end-effector. \( k=1 \).

8. Place the point \( b \) at the intersection of axis \( X_k \) and \( Z_{k-1} \). If they don’t have intersection, place this point at the intersection of \( X_k \) and a common normal to \( X_k \) and \( Z_k \).

9. \( \theta_k \) is the rotation angle from \( X_{k-1} \) to \( X_k \) measured on \( Z_{k-1} \).

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DH Algorithm: Example $\theta – r$ Robot

0. Number the joints from 1 to $n$ beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

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3. Put the origin of $L_k$ in the intersection between axis $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ don’t have intersection, use intersection of $Z_k$ with a common normal of $Z_k$ and $Z_{k-1}$.

4. Put $X_k$ orthogonal to $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ are parallels, then $X_k$ must be perpendicular to $Z_{k-1}$, place it along the link and pointing out.

5. Select $Y_k$ to complete the coordinate system $L_k$.

6. $k=k+1$. If $k<n$, goto 2.

7. Place origin of $L_n$ at top of end-effector. Make coinciding $Z_n$ with vector $a$ (approach), $Y_n$ with vector $o$ (orientation) and $X_n$ with vector normal $n$ of end-effector. $k=1$.

8. Place the point $b_k$ at the intersection of axis $X_k$ and $Z_{k-1}$. If they don’t have intersection, place this point at the intersection of $X_k$ and a common normal to $X_k$ and $Z_k$.

9. $\theta_k$ is the rotation angle from $X_{k-1}$ to $X_k$ measured on $Z_{k-1}$.

10. $d_k$ is the distance from the origin of $L_{k-1}$ to point $b_k$ measured along the $Z_{k-1}$ axis.

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DH Algorithm: Example θ – r Robot

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5. Select \( Y_k \) to complete the coordinate system \( L_k \).

6. \( k=k+1 \). If \( k<n \), goto 2.

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8. Place the point \( b_k \) at the intersection of axis \( X_k \) and \( Z_{k-1} \). If they don’t have intersection, place this point at the intersection of \( X_k \) and a common normal to \( X_k \) and \( Z_k \).

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10. \( d_k \) is the distance from the origin of \( L_{k-1} \) to point \( b_k \) measured along the \( Z_{k-1} \) axis.

11. \( a_k \) is the distance from point \( b_k \) to origin of \( L_k \) measured along \( X_k \) axis.

12. \( \alpha_k \) is the rotation angle from \( Z_{k-1} \) to \( Z_k \) measured on \( X_k \).

13. \( k=k+1 \). If \( k=n \) goto 8.

---

**DoF**

<table>
<thead>
<tr>
<th>( \theta_i )</th>
<th>( d_i )</th>
<th>( a_i )</th>
<th>( \alpha_i )</th>
<th><strong>Home</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( q_1 )</td>
<td>( d_1 )</td>
<td>( 0 ) ( 90^\circ )</td>
<td>( 90^\circ )</td>
</tr>
</tbody>
</table>

\( K=2, \ n=2 \)
DH Algorithm: Example $\theta - r$ Robot

0. Number the **joints** from 1 to $n$ beginning by the base and finishing at yaw, pitch and roll of end-effector (in this order).

1. Assign the coordinate system $L_0$ to base. Axis $Z_0$ coincides with the axis of joint number 1. Then initialize $k=1$.

2. Make coinciding $Z_k$ with axis of joint $k+1$.

3. Put the origin of $L_k$ in the intersection between axis $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ don’t have intersection, use intersection of $Z_k$ with a common normal of $Z_k$ and $Z_{k-1}$.

4. Put $X_k$ orthogonal to $Z_k$ and $Z_{k-1}$. If $Z_k$ and $Z_{k-1}$ are parallels, then $X_k$ must be perpendicular to $Z_{k-1}$, place it along the link and pointing out.

5. Select $Y_k$ to complete the coordinate system $L_k$.

6. $k=k+1$. If $k<n$, goto 2.

<table>
<thead>
<tr>
<th>DoF</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
<th>$a_i$</th>
<th>$\alpha_i$</th>
<th>Home</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$q_1$</td>
<td>$d_1$</td>
<td>0</td>
<td>$90^\circ$</td>
<td>$90^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>$90^\circ$</td>
<td>$q_2$</td>
<td>0</td>
<td>0</td>
<td>$d_2$</td>
</tr>
</tbody>
</table>

7. Place origin of $L_n$ at top of end-effector. Make coinciding $Z_n$ with vector $a$ (approach), $Y_n$ with vector $o$ (orientation) and $X_n$ with vector normal $n$ of end-effector. $k=1$.

8. Place the point $b_k$ at the intersection of axis $X_k$ and $Z_{k-1}$. If they don’t have intersection, place this point at the intersection of $X_k$ and a common normal to $X_k$ and $Z_k$.

9. $\theta_k$ is the rotation angle from $X_{k-1}$ to $X_k$ measured on $Z_{k-1}$.

10. $d_k$ is the distance from the origin of $L_{k-1}$ to point $b_k$ measured along the $Z_{k-1}$ axis.

11. $a_k$ is the distance from point $b_k$ to origin of $L_k$ measured along $X_k$ axis.

12. $\alpha_k$ is the rotation angle from $Z_{k-1}$ to $Z_k$ measured on $X_k$.

13. $k=k+1$. If $k<n$ goto 8.

K=3, n=2  The End !!
Forward Kinematics: \( \theta - r \) Robot

<table>
<thead>
<tr>
<th>DoF</th>
<th>( \theta_i )</th>
<th>( d_i )</th>
<th>( a_i )</th>
<th>( \alpha_i )</th>
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<td>( d_1 )</td>
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<tr>
<td>2</td>
<td>90°</td>
<td>( q_2 )</td>
<td>0</td>
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<td>( d_2 )</td>
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</table>

If we have the value of the joint variables we obtain position and orientation!!
Forward Kinematics: $\theta - r$ Robot
Geometric solution

$\theta_1$ is the rotation angle from $X_0$ to $X_1$ measured on $Z_0$

$\theta_1$ is the rotation angle from $X_0$ to $X_1$ measured on $Z_0$

$d_2$ is the distance from the origin of $L_1$ to point $b_2$ measured along the $Z_1$ axis

planar movement $\Rightarrow p_z = d_1$

$p_x = d_2 \cdot s_1$

$p_y = -d_2 \cdot c_1$

Interpretation of $\theta_1$
Forward Kinematics:
Matlab Robotic Toolbox Functions

Main Functions:

**link**: definition of kinematics (and dynamics) characteristics of different links of a robot using D-H parameters ($\alpha, a, \theta, d$)

**robot**: to concatenate different links and to create the model of the robot

**drivebot**: simplified graphical representation of the robot

**fkine**: to solve the forward kinematics model
Forward Kinematics: $\theta - r$ Robot
Matlab Robotic Toolbox Functions

We are using the “standard” form of D-H algorithm (not the “modified” approach From Craig) $(\alpha, a, \theta, d)$:

$L_1 = \text{link}([\pi/2 \ 0 \ \pi/2 \ d_1])$
$L_2 = \text{link}([0 \ 0 \ \pi/2 \ d_2])$

$\text{robottr} = \text{robot}([L_1,L_2], \text{“theta-r example”})$

$\text{drivebot}($robottr$)$

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Forward Kinematics: $\theta - r$ Robot
Matlab Robotic Toolbox Functions

There are two well known robots already modeled in the Robotics Toolbox:

- **Puma560 (6R)**
- **Stanford (2RP3R)**

You can test the forward kinematics functions using these models
Exercises: Forward Kinematics

Scara (4 DoF)

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Exercises: Forward Kinematics

Robot ABB IRB 6400C (6 DoF)

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Exercises: Direct Kinematics

Robot Mitsubishi RV-M1

![Diagram of Mitsubishi RV-M1 robot with labeled axes and dimensions.]

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Exercises: Forward Kinematics

Stanford Manipulator

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